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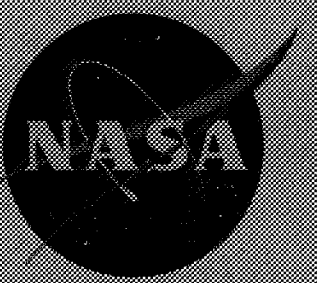

TECHNICAL TRANSLATION

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THE MAGNETIC FIELD OF THE CRAB NEBULA AND ITS CENTRAL STAR

By S. B. Pikel'ner

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THE MAGNETIC FIELD OF THE CRAB NEBULA AND ITS CENTRAL STAR

S. B. Pikel'ner

Resume

1. A Wollaston prism in combination with a 400-mm astrograph was used to measure the polarization at various points in the Crab Nebula (Fig. 1). The results are in general agreement with published data.

2. Assuming a continuous ejection of relativistic particles from the star, it is attempted to determine the field from the form of the spectrum. Radiation within a magnetic field leads to a break in the spectrum (γ is increased by unity). The position of the break determines the field intensity H. The radio-spectrum is constructed from observations; the optical spectrum on the assumption that the slope of the injected spectrum is capable only of increase. The optical spectrum is in agreement with the observed spectrophotometric gradient. At the center the intensity is $H \approx 5 \cdot 10^{-4}$: the average over the nebula is $H \approx 3 \cdot 10^{-4}$.

3. J. Oort and Th. Walraven estimated an average of $H \sim 10^{-3}$ for the nebula on the assumption of equal energy density for the particles and for the field. It is shown that the high value for H is the result of the assumed, arbitrary (generally speaking) energy spectrum. If we take γ from the observed radio spectrum, as I. S. Shklovskiy did earlier, then with the same initial data we arrive at a value, for the center, of $H \approx 6 \cdot 10^{-5}$, in agreement with paragraph 2. This agreement supports the basic premises of the calculation of paragraph 2—the average uniform emission of particles with unaltered spectrum and the insignificant magnitude of dissipation of relativistic particles from the nebula. The optical electrons within the nebula have an energy of $E \approx 3\text{-}4 \cdot 10^{11}$ ev, and their life, considering the structure of the nebula, is about 300 years.

4. The significant difference between the concentration of optical and radio-emission observed toward the center is explained in Ref. 5 by the doubling of the value of H at the center. If we employ the values γ = 1.5 and 2.5, obtained from the observed spectrum and from the discussion in paragraph 2, then this explanation is insufficient. The suggestion has been advanced that a supplemental reason for the sharp

decline in the brightness of the optical rays is the chaotic nature of the field outside its central portion. This follows from observations of the polarization, and is a necessary condition for the retention of relativistic electrons in the nebula. The low speed of diffusion in the chaotic field results in a significant portion of the optical electrons losing energy through radiation, before advancing to the periphery, so that the spectrum in various parts of the nebula differs. Evidently this explains the redder color of the periphery (Ref. 6). The difference in γ for meter and decimeter waves can result in a small difference in the distribution of radio-emission in the nebula.

5. The magnetic field and the relativistic particles press upon the envelope, accelerating its motion. This explains the previously obscure fact of an observed acceleration (Ref. 7). Since pressure and acceleration are known quantities, the mass of the envelope is estimated at $\sim 0.1\ M_\odot$ ($n \sim 2\ cm^{-3}$). Consequently, the greater portion of the mass is not within the filaments, but in the atmospheric portion.

6. The acceleration of the envelope is due to the energy of the particles. Computation is made of the energy of the particles based on the combined action of expansion and radiation losses. A general expression for the spectrum of the particles is obtained. In the spectrum of radioelectrons, K is decreased by γ times, while the spectrum of optical electrons does not change. Taking this factor into account can increase the value of H by no more than 25 percent.

7. The picture of the field is not in agreement with Bachelor's concept of the amplification of H by small scale turbulent motions. However, the field cannot originate again or be thrown off with the envelope. Evidently this is a Galactic field amplified 10-20 times by the motion of streams and other large-scale formations.

8. The fast electrons formed in the star, before entering the nebula, must pass through the star's magnetic field. The period of presence within the field cannot be much less than 1 sec. In order that electrons with $E \sim 4 \cdot 10^{11}$ ev not lose energy, the star's field must be less than 30 oersteds (oer.). Another method of estimating the field is based on the assumption that the electrons passing through the star's atmosphere produce less luminescence than the star itself (16^m). This gives $H < 200$. If it can be shown that the star's fluctuations in luminosity are small, while the relativistic electrons emerge in clusters, then the upper limit of nonthermal emission, and

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consequently also of H, can be lowered.

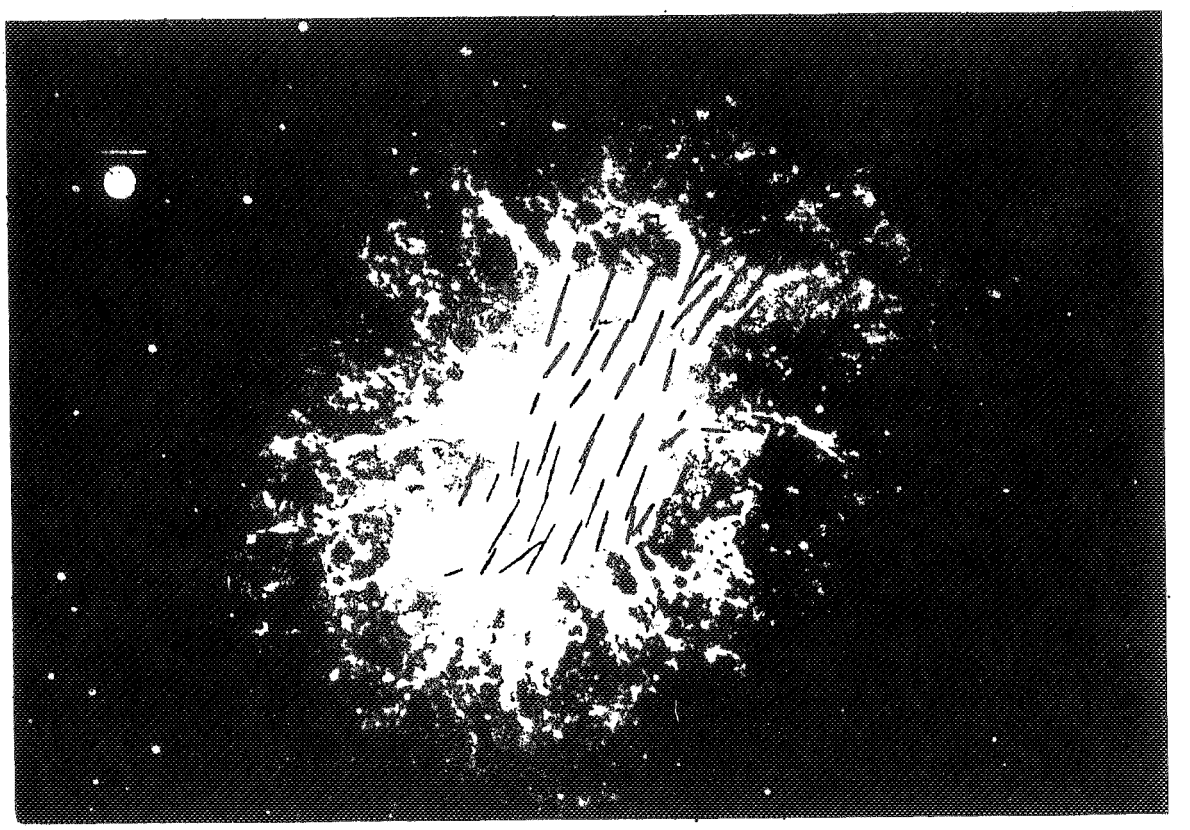

Fig. 1. Polarization at various points in the nebula. On the right: dimension of the measured area and length of segment corresponding to P = 20 percent.

THE MAGNETIC FIELD OF THE CRAB NEBULA AND ITS CENTRAL STAR

S. B. Pikel'ner

1. Observations of Polarization.[1]

The question of the polarization of the optical radiation of the Crab Nebula has recently attracted great attention, since it has made it possible to decide whether this radiation is indeed formed in conjunction with the motion of relativistic electrons within a magnetic field (Ref. 1). The photoelectric measurements of V. A. Dombrovskiy (Ref. 2), and the photographic ones of M. A. Vashakidze (Ref. 3) have revealed the presence of polarization; a detailed picture is given in Refs. 4, 5. A statistical analysis of the question of the variation of polarization at various points is made in Ref. 6. The unequal optical transparency of the objectives of the double astrograph which was used to obtain the photographs did not permit a sufficiently accurate determination of the magnitude and direction of polarization at the various points, except at five points, where it was particularly strong.

In the winter of 1955-1956 the present author and G. A. Shayn made use of a Wollaston prism with angle of divergence of 2° for the study of polarization. The objectives of the collimator and of the polarimeter camera had a focal length of 100 mm; the entire polarimeter was mounted at the focus of the camera of a 400-mm double astrograph (f 1600 mm); the viewing field was limited by a slot whose width was equal to the distance between the images, so that mutual superimposition was eliminated.

With the use of this polarimeter on 20 Nov. and 14 Dec. 1955, 9 photographs on Kodak film 103 a 0 were taken; exposure was 40 minutes, with two positions varying by 45°. The exposures were made by R. Ye. Gershberg. The negatives were calibrated by a tube calibrator and measured on a photoelectric microphotometer (MF-4). The diaphragm

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[1] The investigation of the polarization of the Crab Nebula was conducted in collaboration with G. A. Shayn as a continuation of Ref. 6. G. A. Shayn participated actively in the discussion of the methodology which was used in obtaining the results of paragraph 1. The grave illness and untimely death of G. A. Shayn unfortunately prevented his taking part in the preparation of the theoretical portion of the work.

corresponded to 0.1 mm on the film, or 0'.2. The results of the measurement of polarization were superimposed (Fig. 1) on a photograph of the nebula obtained from Ref. 7. The dimensions of the diaphragm are indicated on the right. The character of the polarization is in general the same as indicated in the results of Refs. 4 and 5, and it supports the conclusions stated in those sources to the effect that in the central portion the polarization is great and quite uniform, while in the peripheral areas it is both smaller and less regular. From the point of view of the hypothesis of relativistic electrons (whose validity, following reliable observation of strong polarization, can be considered as firmly established) this means that the field in the central portion is relatively uniform, and in the peripheral portions it is chaotic.

2. Estimation of the Field from the Spectrum Form.

The comparatively uniform character of the field in the central portion of the nebula renders the statistical mechanism of particle acceleration less probable, since the motions, obviously, cannot be considered chaotic on a small scale. Moreover, it has been shown (Ref. 8) that at the present time conditions within the Crab Nebula are not favorable for particle acceleration. The proposition that relativistic electrons radiating in the luminous portion of the spectrum are residual from an earlier stage of expansion meets with a difficulty indicated by I. M. Gordon (private communication). Indeed, if the nebula is expanding, and if it contains a region with a quasi-uniform field, then the intensity of that field was initially greater, and, consequently, the optical electrons had to be de-excited more quickly. Finally, the most substantial proof that the "optical" electrons are not residual from an initial stage of expansion, but rather are being formed at the present time, has been supplied by J. Oort and Th. Walraven (Ref. 5). They showed that for $H \approx 10^{-4}$ (at such intensity the optical electrons do not have time to be decelerated during the time from the moment of burst) the total energy of the relativistic particles, computed on the basis of their radiation in the radio and optical region (with the assumption that apart from electrons, just as many protons are emitted), must be a thousand times greater than the energy of the magnetic field. In this case the field cannot retain the particle. The total energy of the particles is approximately equal to the energy of the field for $H \approx 10^{-3}$ oersteds. This value is used in Ref. 5 for the Crab Nebula, since with $H > 10^{-3}$ the time of de-excitation of an optical electron t_1 will be too small. With $H \approx 10^{-3}$ the

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optical electrons, according to the calculations of Oort and Walraven, have an average energy of $E \approx 2 \cdot 10^{-11}$ ev. The time of de-excitation for them is about 60 years; however, since a particle spends a considerable part of the time within the peripheral region of the nebula, where the field is weak, the average de-excitation time in Ref. 5 was taken at 180 years.

Short de-excitation time means that the fast particles, as has already been shown, must be under a continuous process of formation at the present time. It is of interest to analyze the data of observations of the Crab Nebula on the basis of the assumption that relativistic particles are continuously formed, and that the number and spectrum of the appearing electrons, as well as the physical conditions (field intensity) have changed little during the second half of the life of the nebula.

It is well known that if electrons make continuous entry into a magnetic field, and if they possess an energy spectrum of $E^{-\gamma}$, then, as a result of de-excitation, a spectrum $E^{-(\gamma+1)}$ is established (Refs. 9, 10). Let us re-derive the conclusion resulting from this position, since for the future we will need to know not only the type of function involving E, but also the values of the coefficients, which normally are not given. Assume that during 1 second $K_0 E_0^{-\gamma_0} dE_0$ electrons are formed with energy E_0 in the interval dE_0, and that $E_0 < E_m$. Because of radiation the electron loses energy (Ref. 10) at the rate

$$-\frac{dE}{dt} = aE^2, \tag{1}$$

where

$$a = 3.8 \cdot 10^{-15} H_\perp^2,$$

if E is expressed in electron-volts. After time t the electron with original energy E_0 will have energy E, which is determined by the relationship

$$\frac{1}{E_0} = \frac{1}{E} - at,$$

whence

$$\frac{dE_0}{E_0^2} = \frac{dE}{E^2}. \tag{2}$$

Electrons with energy E at moment t have been formed at various moments of the preceding time interval

$$t - t_1 = \frac{1}{a}\left(\frac{1}{E} - \frac{1}{E_m}\right). \tag{3}$$

The number of such electrons is /789

$$\begin{aligned} N(E)\,dE &= \int_{t_1}^{t} K_0 E_0^{-\gamma_0} dE_0 dt' = K_0 \int_{t_1}^{t} E_0^{-(\gamma_0-2)} \frac{dE_0}{E_0^2} dt' = \\ &= K_0 \frac{dE}{E^2} \int_{t_1}^{t} \left[\frac{1}{E} - a(t-t')\right]^{\gamma_0-2} dt' = K_0 \frac{dE}{E^{\gamma_0}} \int_{t_1}^{t} [1 - aE(t-t')]^{\gamma_0-2} dt' = \\ &= \frac{K_0}{a} \frac{dE}{E^{\gamma_0+1}} \int_0^{1-(E/E_m)} (1-y)^{\gamma_0-2}\, dy. \end{aligned} \tag{4}$$

With $\gamma_0 \approx 1.0$—1.5 and $E/E_m \approx 10^{-1}$—10^{-3} the values of the integral I are contained within the interval 1.4—6.9. If $E \ll E_m$, then $I = 1/(\gamma_0 - 1)$. Thus, as a result of de-excitation, the energy spectrum $KE^{-\gamma}$ is established, where $K = \frac{I}{a} K_0$ and $\gamma = \gamma_0 + 1$. In Ref. 5 it was assumed that $N(E) = K_0 E^{-\gamma_0} (aE)^{-1}$ which will give a small error. If the age of the system is limited, i.e. electrons appear during the time T, (for the Crab Nebula $T \approx 900$ years) the break in the spectrum starts from the energy $E_1 = (aT)^{-1}$. The region of break is comparatively narrow—approximately from $1/2\ E_1$ to $2E_1$, and outside of it the spectrum is very little different from that with constant γ. For $E < E_1$ the spectrum has the initial form and the number of particles

$$N(E)\,dE = K_0 T E^{-\gamma_0}\,dE.$$

The quantity E_1 is a function of a—i.e., of H. Therefore, if we determine E_1 from observations, it is possible to calculate H. Since the energy spectrum of electrons determines the spectrum of their radiation, the quantity E_1 may be determined from the point of the break in the spectrum of the observed radiation. According to Refs. 9 and 10, the radiation flux from a source at a distance R and containing $KE^{-\gamma}dE$ electrons with energy E—E + dE, is equal to (per unit frequency)

$$F_\nu = \frac{K}{R^2} 1.3\cdot 10^{-22} (2.8\cdot 10^8)^{\frac{\gamma-1}{2}} H^{\frac{\gamma+1}{2}} \lambda^{\frac{\gamma-1}{2}} U(\gamma)\ \text{erg/cm}^2\ \text{sec. cycle} \qquad (5)$$

The radiation of each electron is maximum at the frequency

$$\nu_{\max} = 5{,}4\cdot 10^{-6} H_\perp E^2 \quad (E \text{ в eV}) \qquad (6)$$

and becomes insignificant whenever E is varied by a few unit factors from the optimum value for a given frequency. Therefore, if γ is not constant, but varies uniformly over the spectrum, expression (5) may be utilized when K and γ correspond to an average value for the effective interval of energy E.

The spectrum of the radio-emission may be reconstructed from published data on the flux at various frequencies (Fig. 2). The values of F_ν for λ = 3.2 and 10 cm are given in accordance with Refs. 11, 12; the others from a summary of Pawsey (Ref. 13) and Refs. 14, 15. Beginning with λ = 2m, the spectrum decreases in such a manner that $F_\nu \sim \lambda^{0.25}$ (γ = 1.5). The flux in the optical range (λ = 4250 Å) can be obtained from data of Baade (Ref. 7)—the total radiation in all

directions is equal to $3.4 \cdot 10^{21}$ erg/sec cps. However, it is important to take into account that the distribution of radio-emission and luminous radiation in the nebula is variable, and that the latter shows a significantly stronger concentration toward the center. We must obtain a complete spectrum of some uniform portion of the nebula, say close to the center. The ratio of emission at $\lambda = 300$ cm to that of emission at $\lambda = 4250$ Å has been estimated in Ref. 5, and for the sake of comparison of results we have taken the same value—230. This gives the position of the point $\lambda = 4.25 \cdot 10^{-5}$ in Fig. 2, which now represents the spectrum of the center with accuracy up to a constant coefficient. /790

The variation of the spectrum in the optical portion can be reconstructed if we begin from the data of Minkowsky (Ref. 16) on spectrophotometric gradients free from the influence of absorption; this is taken to be $1^m.2$ within the photographic region. From these data it is possible to determine that for $\lambda = 4500$ Å, $I_\lambda \infty \lambda^{0.81}$ ($\gamma = 2.62$), and for $\lambda = 6000$ Å, $I_\lambda \infty \lambda^{0.59}$ ($\gamma = 2.18$). The error in absorption of $0^m.3$ shifts the exponent by 0.2—that is, shifts λ by 0.4.

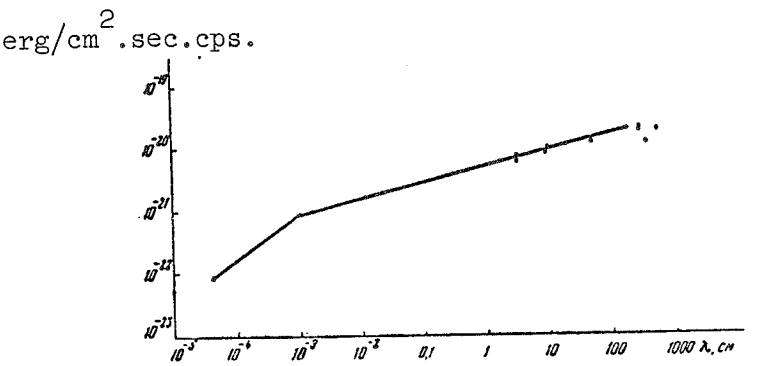

Fig 2. Spectrum of the radiation from the Crab Nebula. The radio-spectrum is given on the basis of observational data; the point of the optical spectrum is based on the assumption that the optical radiation of the central region is 230 times weaker than the radio-emission. The slope of the optical spectrum is based on the assumption that γ has been altered by not less than one unit.

The uncertainty in the variation of the optical spectrum would seem at first glance to render the determination of the point of break useless. However, if we consider certain peculiarities of the energy spectrum, we may make rather definite conclusions. We shall start off from the fact that with increase in energy, the curvature of the spectrum can only increase (this follows from a comparison with cosmic rays), and that in a rather narrow interval of the spectrum γ must change by unity, i.e., the slope of the line must change by 0.5. For the time being we shall assume that the initial spectrum is characterized by a value of $\gamma = 1.5$ (obtained from the radio-spectrum) up to optical energies, so that the reconstructed spectrum in this region has $\gamma = 2.5$. Such a slope is in agreement with the variation of the optical spectrum and gives a point of intersection of about $\lambda_1 = 10^{-3}$ cm. If the initial and the corresponding re-derived spectra in this area are steeper, then the point of intersection is displaced to the side with shorter wavelengths. The assumed ratio of intensity of radio-emission at the center to the optical radiation (230) may well be in error by a factor of 2, and this would result in roughly the same error in wavelength at the point of intersection. In summing all that has been said and considering the possible errors in the construction of the radio-spectrum, we may come to the conclusion from Fig. 2 that the point of break has $\lambda < 2.10^{-3}$cm, but more likely $\leqslant 10^{-3}$ cm. Now we can estimate the field intensity. From Ref. 6, assuming $\nu_m = 3\cdot10^{13}$ ($\lambda = 10^{-3}$cm), we find that $E_1\sqrt{H_\perp} = 2.3\cdot10^9$. If we replace $H_\perp$ by the average field intensity in the nebula, we may assume that $E_1\sqrt{H} \approx 2.10^9$. On the other hand, from the equality $E_1 = (aT)^{-1}$, where T = 900 years, it follows that $E_1^2 H = 10^4$. Comparing the two expressions, we find $H \approx 3\cdot10^{-4}$, $E_1 \approx 10^{11}$ ev. The allowed variations in the position of the point of break have only an insignificant influence on the quantity $H(H \sim \lambda_1^{1/3})$. The field in its central area can be considered as roughly equivalent to $5\cdot10^{-4}$. If the optical spectrum corresponds to $\gamma > 2.5$, then H will be somewhat less.

3. Estimation of the Field from Energy Density /791

In Ref. 5 it is assumed that the lower boundary of the field is

10^{-3}, since any lesser value of intensity would imply that the energy density of the particles exceeds the energy density of the field. However in our case the values of γ are different from those assumed in Ref. 5, and therefore the calculation of the energy density of the particles must be made anew. In Ref. 5 the variation of the energy spectrum (the quantity γ) was determined not from the radio-emission spectrum, but rather from a comparison with the energy spectrum of cosmic rays. At first γ for forming particles was taken to be the same as for the primary cosmic rays—0.8, 1.8, and 2.8 in the energy intervals from 0.155 to 1.55 Bev, from 1.55 to 15.5 Bev and > 15.5 Bev respectively. In addition, for more rapid particles there was taken into account a decrease in the life because of de-excitation, this being equivalent to a change in γ by one unit. However, the ratio of radio-emission to optical radiation as calculated from these values is significantly less than the observed value. To bring theory into line with observations, all of the values for γ were arbitrarily lowered by 0.5. This is the weak point in Ref. 5: if the spectrum is to be arbitrarily altered, then in general it will be impossible to relate it to cosmic rays. The Crab Nebula is evidently not a typical object, as is evidenced by its flattish radio-spectrum. Therefore, a determination of γ on the basis of radio-emission from the nebula such as has been made in Refs. 17, 18 is more reliable.

In Ref. 5 the total energy of particles emitted from the star is determined during the entire life of the nebula; and it is assumed that as many protons as electrons were formed. Generally speaking, it is necessary to take into account the total energy of all presently-existing particles; but in the case of protons it is a matter of the total energy of all protons which have been emitted, since they lose but little energy. For the electrons, this involves an error factor of 2—3. A comparison of the energy of the field and of the relativistic electrons has been made by I. S. Shklovskiy (Refs. 17, 18) on the basis of the assumption that H is equal to 10^{-4} or several times that value. Since he did not take into account the protons; evaluating the total energy only on the basis of electrons presently existing, he concluded that the energy density of the field and of the particles was of the same order of magnitude. In Ref. 17, while the variation of the spectrum is considered, the structure of the nebula itself is not taken into account. Let us repeat this calculation for the central portion of the nebula, and for comparison with the data of Ref. 5 determine the total energy of all emitted particles, assuming that protons are given off in the same numbers as electrons. Here the upper boundary of energy is of interest to us, and therefore, we shall consider that the entire spectrum of emitted particles is characterized by $\gamma = 1.5$. The flatter variation of the spectrum in the area of the metric waves, and perhaps also the steeper

variation of the initial (not revised) spectrum in the optical range, only lowers the total energy of the particles.

In order to calculate the energy density of the particles one must know the luminescence of a unit of volume of the nebula at any given frequency in the radio range. Let us take $\nu = 160 \cdot 10^6$ ($\lambda = 187$ cm), where the point corresponding to the flux lies on a straight line, representing the spectrum (Fig. 2). Assuming a value of 230 for the ratio of radio-emission to optical radiation at the center, we may calculate from the same data as used in Ref. 5 that 1 cm^3 of space within the central zone of the nebula radiates approximately $\lambda = 187$ cm $J'_\nu = 10^{-31}$ erg/cm^3sec cps.

From Ref. 5 it is easy to determine that the radiation in all directions is

$$J'_\nu = 4\pi R^2 F'_\nu = 4\pi K' \cdot 1.3 \cdot 10^{-22} (2.8 \cdot 10^8)^{\frac{\gamma-1}{2}} H^{\frac{\gamma+1}{2}} \lambda^{\frac{\gamma-1}{2}} U(\gamma), \qquad (7)$$

where the accent marks indicate that the quantities J_ν, K, F_ν relate to 1 cm^3 at the center of the nebula. Assuming $\gamma = 1.5$, $U(\gamma) = 0.2$, $H = 10^{-3}$, we find $K' = 3.6 \cdot 10^{-9}$ CGS.

The total energy of electrons released per 1 cm^2 is equal to /792

$$\int_{E_2}^{E_3} K' E^{-\gamma+1}\, dE = \frac{K'}{2-\gamma} (E_3^{2-\gamma} - E_2^{2-\gamma}) \approx 5 \cdot 10^{-9} \text{ erg/cm}^3 \qquad (8)$$

if we assume that $E_3 \approx 0.5$ ergs $= 8 \cdot 10^{11}$ ev; this exceeds by four times the average energy of the optical electrons. Assuming that the energy of the emitted relativistic protons is equal to that of the electrons[1], we arrive at an upper boundary for energy density of the

[1] See next page for footnote.

fast electrons of 10^{-8} erg/cm^3—a value 7.3 times less than that of Ref. 5 and 4 times less than the energy density of the field $H \approx 10^{-3}$. The energies are approximately equal ($\sim 1.5 \cdot 10^{-8}$ erg/cm^3), for $H \approx 6 \cdot 10^{-4}$, which agrees with the quantity obtained earlier for the center of the nebula from the position of the point of break. The total energy of fast particles emitted since the explosion of the supernova amounts to $1.5 \cdot 10^{-8} \cdot 4 \cdot 10^{54} \cdot 8.3 = 0.5 \cdot 10^{48}$ ergs—which is somewhat less than the energy emitted in the explosion itself, and 6 times less than that quoted in Ref. 5. This considerably reduces the difficulties connected with the energy aspect of the problem, which were discussed in Ref. 5.

In summary, we may say that if the course of the energy spectrum is determined from radio data, then the magnitude of the field intensity as obtained from various methods is equal, at the nebular center, to $5\text{-}6 \cdot 10^{-4}$, with nebular average of $H \approx 3 \cdot 10^{-4}$. In such a field the optical electron has an average energy of about $3 \cdot 10^{11}$ to $4 \cdot 10^{11}$ ev; loses half of it in about 300 years; in the central region loses half of it in about 100 years. The energy density of the particles under such intensity is approximately equal to the magnetic energy density. The agreement of results obtained by two different methods is evidence of the fact that the basic premises of the calculation are sound—the particles are emitted, on the average, uniformly and with a constant spectrum, while the dissipation of particles from the nebula is negligible.

4. Distribution of Optical and Radio-Emission.

If the field over the whole nebula were almost uniform, the electrons would rapidly move out along the lines of force. The insignificant magnitude of the actual dissipation is explained, clearly, by the fact that in the more peripheral portions of the nebula the field becomes considerably more perturbed (this is supported in Ref. 5 on the basis of polarization data), so that the speed of diffusion is greatly diminished.

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1 Since the Crab Nebula is evidently not a typical object, we can say nothing of the spectrum of forming protons. It may be that there are much fewer of them (spectrum with $\gamma = 2$); then the total energy in the field will also be much less.

Closely associated with the question of the diffusion of electrons, is that of the distribution of optical and radio-emission within the nebula. In Ref. 5 the observed concentration of optical radiation towards the center of the nebula is less than that of radio-emission. According to data of Singer (Ref. 5) the ratio of scales is 1.69 (and this agrees approximately with the data of eclipse observations), was explained by the fact that the intensity of the field at the central portion is some two times greater than on the periphery. Indeed, according to Ref. 5, $F_\nu \approx H^{(\gamma+1)/2}$, and since γ in the optical spectrum is greater, the degree of concentration is also greater. It was assumed that the concentration of particles at the center is twice that of the concentration at the periphery. This is very probable, since the electrons diffuse only along H, forming as it were a one-directional distribution. Then the concentration of electrons must be proportional to the density of the lines of force; that is, proportional to H. Agreement with Ref. 5 was satisfactory up to r = 1'.5 from the center, but beyond this point optical radiation dropped off too fast.

In our case this explanation is inadequate, since the magnitude of γ in the radio region is larger than in Ref. 5; while in the optical area it is less. Therefore in our case the dependence of relative brightness upon H is less. The difference in γ by one unit /793 means that the ratio of luminescence is proportional to $H^{0.5}$, with $J'_\nu \propto H^{2.25}$ in the radio range, and $J'_\nu \propto H^{2.75}$ in the optical range, if the concentration of electrons $\propto$ H. The concentration of the radio-emission towards the center can be explained by the increase of H in the central portion by a factor of 2, but here the concentration of the optical radiation will be less than indicated by the observations, even if we assume that $\gamma = 3$.

We may attempt to explain the discrepancy by the action of electrons emitted directly from the star. This action may be substantial if the time of diffusion through the central region (t_2) is not small a portion of the time of de-excitation, t_1. If during 1 ond $K_0 E^{-\gamma_0} dE$ electrons with energy E are emitted from the star, en within the central region there will be $t_2 K_0 E^{-\gamma_0}$ electrons leaving the star directly. The number of electrons with the same

energy level in the entire nebula will be, in accordance with (4),

$$\frac{K_0}{a} E^{-(\gamma_0+1)} dE \int_0^{1-E/E_m} (1-y)^{\gamma_0-2} dy \approx 2\frac{K_0}{dE} E^{-\gamma_0} dE = 2t_1 K_0 E^{-\gamma_0} dE. \quad (9)$$

If the electrons were uniformly (∞ H) distributed within the nebula, then in the central region there would be a portion of them, determined by the time of presence of an electron in the central region. According to the estimate of Ref. 5, $a \approx 1/8.3$. The relative surplus of optical radiation in the central portion with incomplete intermixing is $t_2/2at_1$. The quantity t_2 depends upon the average rate of diffusion. Along H the electrons move with a speed of about 1/2c; but even in the center the field is not absolutely uniform. We can make a rough estimate of the rate of diffusion based upon the magnitude of average polarization in the central portion of the nebula—less than 20 percent. This means that the average magnetic flux in this area amounts to 20 percent of the flux which would be present in a uniform field of the same intensity. If this decrease in average flux is the result of a partial perturbation in the field, then the magnitude of the decrease must be approximately equal to the by-pass factor, which in the present instance must be close to 5, so that the rate of diffusion in the central portion may be taken as equal to 0.1 c. The data offered by Baade in Ref. 5 concerning the motion of luminous formations—clusters of relativistic electrons—yield a velocity of the order of 25,000 km/sec, or just about 0.1 c. The time of motion within the central zone (r = 1') with such velocity is $t_2 \approx 3 \cdot 10^8$ sec, or 10 years. The quantity $t_1 \approx 300$ years, so that the relative excess of radiation is ≈ 0.14. However, beginning with $r \approx 0.'8$, the magnitude of polarization begins to fall off rapidly—in other words, the field becomes more perturbed. The very fact that the substantial part of the relativistic electrons has been retained within the nebula for a period of 900 years indicates that during 300 years of their life the optical electrons will not have sufficient time to diffuse through the entire nebula. If the average velocity of diffusion in the outer portions is 0.02 c, then the relative increase in brightness at the center, compared with the case of uniform (∞H) will be 0.7—i.e., quite marked. Refinement of this quantity for the case of an indefinite structure is meaningless, but the calculation nevertheless shows that the effect in question may in principle explain the observed excess of optical radiation in the central portion and the rapid falling off of optical brightness in the external layers.

We should note that the spectrum of the "new" electrons has γ a unit less than that of all the optical electrons of the nebula. Consequently, the radiation of the center must be more blue, and that of the periphery more red, than average radiation. It is possible that this explains a certain difference between the color of the periphery and of the center as noted in Ref. 6. It is interesting to verify whether the color of the portions observed by Baade was bluer than the nebula. It is easy to calculate that, if to radiation having a spectrum characterized by γ_1 we add radiation with $\gamma = \gamma_1 - 1$ and consisting of β part of the intensity, the total radiation will have $\gamma = \gamma_1 - \beta/(1+\beta)$. If $\beta > 0.5$, the variation in γ may be significant. /794

This effect must be kept in mind if we are to determine the point of break in the spectrum, proceeding from the observed optical spectrum of the center of the nebula.

Still another circumstance may be pointed out. Since the spectrum of the radio-emission obviously has $\gamma = 1.5$ for $\lambda < 1.5$ m, and $\gamma = 1.0$ for $\lambda < 2$ m, the radiation of the corresponding wavelengths must exhibit a varying concentration towards the center. Possibly this explains the discrepancy between the data of Baldwin and Singer (Ref. 5) and the discrepancy between Refs. 19 and 20 on the one hand, and Ref. 21 on the other ($\lambda \approx 3$ and 6 m, and $\lambda = 1.8$ and 0.33 m, respectively).

The fact noted in Ref. 19 of the noncoincidence of the optical and radio-emission centers of the nebula (the latter is displaced by 1'.5 to the SE) means that the peripheral portion with weaker field possesses a large extension in the SE direction.

5. Estimating the Mass of the Envelope.

Since the luminescence of the amorphous portion of the nebula is produced by the motion of relativistic electrons, measurement of the brightness does not give us a direct indication of the mass. The radiation of the filaments was used earlier to estimate their mass—of the order of several hundredths of the mass of the sun (Ref. 1). It is of interest to make an estimate of the total mass thrown off with the explosion of the supernova. At one time when it was believed that the field of the nebula was increasing from turbulences whose velocity, based on observations of local portions, was taken to be 300 km/sec; the condition of the equivalence of the magnetic and the kinetic energies implied a concentration of atoms with value of $n \sim 1$—that is, a mass of $\sim 10^{32}$ g ~ 0.05 M (Ref. 17). However, a weakly perturbed—and locally quite uniform field (Ref. 22)—which presents

a separate problem, renders this method somewhat unreliable, although the presence of even a partially irregular field requires the presence of matter with a comparable density of kinetic energy. In estimating the mass, we may make use of the fact that the magnetic field and the relativistic particles present in the nebula react upon the envelope, accelerating the motion of its external layers and braking the motion of its internal layers. This is the only way of interpreting the fact of the presence of matter close to the star, although this matter was thrown off 900 years ago at great velocity; and also the fact of the accelerated expansion of the nebula (Ref. 7). Actually, Baade has calculated from Duncan's measurements (Ref. 23) that if the nebula had been expanding along its major axis with its present velocity, the explosion must have taken place 758 years before the measurements referred to: whereas, from 1054 to 1939 is a gap of 885 years. Assuming the acceleration to be constant, the value of the latter was found to be 0.0011 cm/sec^2 (Ref. 7). The radiation pressure could not produce such an acceleration, and its presence was an unresolved difficulty which can be removed only by entirely new concepts on the nature of the Crab Nebula. Since the density of the energy of cosmic rays is of the same order as the density of the magnetic energy, the difference in pressure inside and outside the nebula is of the order of $2\frac{H^2}{8\pi} \approx 10^{-8}$ d/cm^2. Such a force, acting upon a cylinder 1 cm^2 in cross-section area and about $2 \cdot 10^{18}$ cm in length would impart an acceleration of 0.0011 cm/sec. From this it follows that the mass of the cylinder would be $\sim 10^{-5}$ g and its density $5 \cdot 10^{-24}$ g/cm^3—that is, $n \sim 2$. Consequently, the total mass of the nebula is 0.1 $M_\odot$, as in the estimate quoted earlier. The mass of the filaments, evidently, is several times less than the mass of the amorphous portion.

The magnetic field on the average is directed along the minor axis of the nebula. The action of the magnetic field in this case may amount to a braking (tension tends to shorten the line of force) which is compensated by the pressure of the relativistic particles. It is possible that the shorter radius of the nebula here is explained by precisely this circumstance.

6. Influence of the Expansion of the Envelope on the Spectrum of the Particles. /795

If the pressure of the relativistic particles produces an acceleration of the envelope and an increase in kinetic energy, then

the energy of the particles must itself be diminished. Since energy decrease proceeds according to a law different from that for radiation losses in a magnetic field, it is possible to establish a spectrum different from that accepted earlier in this article. For this reason, let us make a detailed calculation. The expression for the loss of energy by particles moving in an envelope expanding with velocity V has been derived in Ref. 8:

$$\frac{dE}{dt} \approx \left[u^2 - \frac{avVl}{r}\right] \frac{v}{lc^2}, \qquad (10)$$

where $a \approx 1$, u is the velocity of the chaotic motions of gas formations within the envelope, l is the dimension of such formations, v is the velocity of the particle (in our case $v \approx c$), and r is the radius of the envelope. The first member of (10) determines the acceleration of a particle by Fermi's mechanism; the second, the braking during the expansion of the envelope which is analogous to an adiabatic cooling of the gas. It is shown in the same source that in the very first period following the explosion acceleration may occur, but that under present conditions braking is significantly more dominant. Let us estimate the upper limit of the effect of braking, leaving out the first member and substituting a = 1. Then all of the individual characteristics of the motion are cancelled out, and we have only the simple expression

$$\frac{dE}{dt} = -\frac{V}{r}E = -\frac{1}{t}E, \qquad (11)$$

where t is the time elapsed from the moment of the explosion (at the present moment $t = T = 900$ years). In fact $r/V \approx 770$ years (Ref. 7), but a calculation of the acceleration of the envelope will yield an effect of second order, and we shall assume the expansion to be uniform. We shall assume that (11) is applicable only when $t > t_2$, when braking of particles has set in. Let us note that losses due to radioemission are not taken in account in (11). If we include these, then the energy loss will be equal to

$$\frac{dE}{dt} = -\frac{1}{t}E - aE^2. \qquad (12)$$

For the case when, at the time t', the energy of a particle is E',

the integral of equation (12) has the form:

$$\frac{1}{Et} - \frac{1}{E't'} = a \ln \frac{t}{t'}. \qquad (13)$$

Proceeding as in section 2, we arrive at an expression for the spectrum at time t:

$$N(E)\,dE = K\int_{t_1}^{T} E'^{-\gamma} dE'\,dt' = \frac{K}{T}\,\frac{dE}{E^2}\int_{t_1}^{T} t'^{\gamma-1}\left(\frac{1}{ET} - a \ln \frac{T}{t'}\right)^{\gamma-2} dt', \qquad (14)$$

where the lower limit of the integral is to be interpreted as the time from which particles could be preserved up to the present time. Particles which possessed the energy $E' = \infty$ at the initial period of deceleration ($t' = t_2$) now possess energy E_1, given by the equation

$$\frac{1}{E_1} = aT \ln \frac{T}{t_2}. \qquad (15)$$

For $E < E_1$, $t_1 = t_2$; that is, the particles possessing such energy could be preserved from the early period following the explosion. For $E > E_1$, t_1 is given by the equation

$$\frac{1}{E} = aT \ln \frac{T}{t_1}. \qquad (16)$$

It is easy to show that for $E \ll E_1$ the second term under the integral sign may be neglected. Then, following integration, since $T \gg t_2$, we have /796

$$N(E) = T\,\frac{K}{\gamma} E^{-\gamma}. \qquad (17)$$

Consequently, our calculation of the expansion of the envelope does not alter the form of the spectrum of the radio-electrons, but it does lower the factor K by $\gamma = 1.5$ times.

Now let us consider the spectrum of the optical electrons. Transforming (14), we find, (assuming that $T-t_1 \ll T$),

$$N(E) = \frac{K}{T^{\gamma-1}E^{\gamma}} \int_{t_1}^{T} t'^{\gamma-1}\left(1 - aET\ln\frac{T}{t'}\right)^{\gamma-2} dt' =$$

$$= \frac{K}{T^{\gamma-1}E^{\gamma}} \int_{0}^{T-t_1} (T-t)^{\gamma-1}(1-aEt)^{\gamma-2} dt = \qquad (18)$$

$$= \frac{K}{aE^{\gamma+1}}\left(\frac{1}{\gamma-1} - \frac{1}{\gamma-1}[1-aE(T-t_1)]^{\gamma-1} - \right.$$

$$\left. - \frac{1}{aET}\left\{\frac{1}{\gamma} - aE(T-t_1)[1-aE\cdot(T-t_1)]^{\gamma-1} - \frac{1}{\gamma}[1-aE(T-t_1)]^{\gamma}\right\}\right).$$

In the same approximation we find that $aE(T-t_1) \approx 1$ and $aET \gg 1$.

Substituting in (18), we have

$$N(E) = \frac{K}{(\gamma-1)aE^{\gamma+1}}, \qquad (19)$$

In other words, the spectrum is just the same as if there were no effect of expansion (4). This might be expected, since expansion, as was apparent from (13), has little influence on particles formed comparatively recently.

Thus, in order to compensate for the effect of the expansion of the envelope, we should lower the curve of the radiospectrum in Fig. 2 by an amount corresponding to $\gamma = 1.5$. This will move the point of break in the spectrum to the right; that is, it will increase H, but only insignificantly—by 25 percent—and in view of the fact that the upper limit of the effect was evaluated without taking into account statistical acceleration from internal movements, the results of paragraph 2 remain valid.

7. The Nature of the Field

The comparatively uniform character of the field in the

different portions of the nebula, and the presence on the photographs (Ref. 23) of streams (field tubes) are evidence against the strengthening of the field by small-scale turbulences, as conceived by Batchelor (Ref. 24). However, we may not exclude entirely the hypothesis that the nebular field is strengthened by small-scale motions of the Galactic field. Motions consisting of separate streams and clusters are able to form a long field tube with intensity comparable with $H \approx v\sqrt{4\pi\rho}$, as in a magnetohydrodynamic wave. The presence of separate dense clusters along the path of expansion of the envelope may also create a field in the form of elongated protruberances directed towards the star. In any event, the amplification of the Galactic field by several factors of 10 is a possibility more easily interpreted than the formation of a magnetic flux upon an explosion, or other such hypotheses. Neither could the field be thrown off from the star, since its magnetic flux is too great, indeed more than that possible in the star. Chandrasekar's estimate of the upper limit of the field of the star, quoted in Ref. 5, involves a curious error in dimensionality—the total gravitational energy of the star and the density of magnetic energy are equated. Therefore, an enormous value for H is obtained. With the "spread" of this field into the nebula, /797 another error is permitted which compensates for the first—actually the flux of the field is preserved rather than its energy. As a result, a value close to 10^{-3} was obtained for H within the nebula, this being explained simply by the fact that the total energy freed upon the explosion of the supernova (in comparison with the magnetic energy of the envelope) is close to the gravitational energy of the star.

8. The Magnetic Field of the Star.

The ineffectiveness of the statistical mechanism in explaining the acceleration of particles within the nebula at the present time, and, outstandingly, the observations of Baade which have shown that the luminous clusters of relativistic electrons are moving away from the star, prove that high-speed particles are being emitted by the star. If this is true, they must pass through the star's magnetic field and be partially de-excited, as has been indicated by I. M. Gordon, who in a number of writings has examined the possible role of relativistic electrons in explaining chromospheric explosions, nova and supernova explosions, and phenomena in variable and erratic stars (Ref. 25). An analysis of this situation can give information about the magnetic field in those portions of the star's atmosphere from which the particles are being emitted. An estimate of the field is of great importance in the present instance, since, first of all,

it gives some sort of basis for adapting the theory of relativistic electrons to the processes within erratic stars; and, secondly, the very process of the formation of relativistic electrons can be associated with the electromagnetic processes. In any case, it would seem to be no accident that the active areas on the sun, which in many respects recall erratic stars (Refs. 26, 25) are usually associated with spots.

The observations of the Crab Nebula show the presence of electrons with energy $E \approx 4 \cdot 10^{11}$ ev. If these electrons leave the star, it means that they have been in its magnetic field for a period less than the time of de-excitation $t_1 = (aE)^{-1}$, following their formation. It follows from this that the time of stay within the atmosphere is $t < 600 H_*^{-2}$. If the field of the star $H_* \approx 10$ oersteds, $t < 6$ sec, which, generally speaking, is admissible, since, if the velocity of diffusion of an electron is $\approx 1/3$ c, then during 6 seconds it will traverse a distance almost equal to the radius of the sun. However, even the value $H_* \approx 30$ oersteds seems to be too large, because even at the very high diffusion velocity assumed, an electron would travel only 60,000 km from the star, and this is, obviously, less than the characteristic dimension of the field. If we assume that the relativistic electrons are not diffused from the star, but rather thrown out with matter with a velocity of close to 1,000 km/sec, then the conditions become still more rigorous, so that the field within the ejected mass must be quite weak if the electrons are not to lose energy during the time of the ejection.

We may advance another and independent method for estimating the field of the star, a method which narrows the possible boundaries. When the relativistic electrons pass through the field of the star, a portion of them will radiate in the visible portion of the spectrum. Since the number of electrons of different energies formed on the average per unit of time is quite well known, just as is the luminosity of the star—that is, the upper limit of nonthermal radiation—we may estimate the upper limit of the field intensity, assuming that the electrons remain within the atmosphere of the star for several seconds.

In Section 3 it was calculated that for $H = 10^{-3}$, $K' = 3.6 \cdot 10^{-9}$ CGS per 1 cm^3. With $H_y = 5 \cdot 10^{-4}$ this will result in, (according to

(7)), $K' = 8 \cdot 10^{-9}$ CGS. The quantity K characterizes the concentration of electrons emitted from the star after 900 years (not taking into account the de-excitation), per 1 cm^3 of nebular space. In order to relate this to the entire nebula, it is necessary to multiply K by $0.4 \cdot 10^{55} \cdot 8.3 = 3.3 \cdot 10^{55}$; and to express the number of electrons emitted per second, we must divide by the time since the explosion occurred—that is, by $2.8 \cdot 10^{10}$ sec. As a result we find that /798 $K''E^{-1.5}dE$ electrons with energy E are emitted from the star during 1 second: here $K'' = 8.10^{-9} \cdot 3.3 \cdot 10^{55} (2.8 \cdot 10^{10})^{-1} = 0.9 \cdot 10^{37}$ CGS. If electrons are retained in the atmosphere for t seconds, then their number is characterized by the quantity $tK''E^{-1.5}dE$. The radiation from these electrons is

$$J_\nu^* = 4\pi t K'' \cdot 1.3 \cdot 10^{-22} (2.8 \cdot 10^8)^{0.25} H_*^{1.25} \lambda^{0.25} \cdot 0.1 \text{ erg/sec cycle} \quad (20)$$

The optical electrons emitted from the star accumulate within the nebula in the course of the period of de-excitation $t_1 \approx 300$ years $\approx 10^{10}$ sec divided by $\gamma - 1$ (See equation 4). The total radiation in a narrow spectral interval is

$$J_\nu = 4\pi \cdot 2t_1 K'' \cdot 1.3 \cdot 10^{-22} (2.8 \cdot 10^8)^{0.25} H^{1.25} \lambda^{0.25} \cdot 0.1 \text{ erg/sec cycle} \quad (21)$$

where γ is taken to be 1.5, because de-excitation is calculated from the quantity $t_1/\gamma - 1$. In the center there is a proportion of 1/8.3 of the electrons; for this reason K'' for the center is 8.3 times less. The ratio of nonthermal radiation of the star and of the center ($r \leqslant 1'$)

$$\frac{J_\nu^*}{J_\nu} = \frac{8{,}3\, t H_*^{1.25}}{2t_1 H_\odot^{1.25}} < \frac{1}{630 \cdot 0.34} \quad (22)$$

can be obtained from the difference between the stellar magnitude of the star (16^m) and that of the nebula (9^m), and from the condition that the center radiates 0.34 of the total light of the nebula. Substituting in (22) $H_u = 5 \cdot 10^{-4}$, $t \geqslant 1$ sec, we find H to be less than 200. Consequently, the second method of estimating the field affords no new information and merely reiterates the first. However, if observations show that the luminosity of the star is constant within, say, a limit of 10 percent, with indications of nonuniform emission of electrons on the basis of Baade's data, the proportion of nonthermal radiation from the star may be lowered, along with the upper limit of the star's field. In conclusion we may say that the former supernova of 1054 is evidently without a field of any force.

Possibly this is the reason why of all the radio-nebulae—remnants of supernovae observed within our galaxy, only the Crab possesses a noticeable continuous spectrum in the optical range; although, for example, Cassiopeia-A is a more powerful source of radio-emission. If a star has $H \approx 50$-100, this is sufficient for electrons with optical energies to be absent from the nebula. Moreover, if the field of the star is of the order of several hundred oersteds, then the radio-electrons ($E \approx 10^9$ ev) also lose energy in the atmosphere. Thus the fields of other former supernovae associated with sources of radio-emission are evidently included within the 50-200-oersted interval if we consider the nebular field to be close to 10^{-3}—$3 \cdot 10^{-4}$. In a number of such cases it is possible to consider observing the nonthermal radiation of the star.

The spectrum of the Cassiopeia-A source, and of other such objects, is distinguished from that of the Crab Nebula not only by the absence of optical radiation, but also by a steeper variation of the radio-spectrum ($\gamma = 3$). This peculiarity may well be innate in the mechanism of particle-acceleration itself, but can be interpreted for the same mechanism. Let us suppose that particles are being generated with $\gamma = 1.5$-2, and that the field of the star in one region or another from which they are being emitted is of sufficiently complex structure that some particles are emitted soon after formation, while others remain with the star a longer period. If the field intensity is such that the time of de-excitation of the electrons corresponding to the radio-emission of the nebula ($E \approx 10^9$ ev) is close to the minimum time during which an electron can escape, then the electron spectrum within the stellar atmosphere is altered (γ is increased by unity), and the electrons with such altered spectrum will be dissipated from the

star to fill the nebula.

Still another remark is in order. Since electrons lose their energy rapidly in a magnetic field whose intensity in the case of erratic stars can hardly be less than several oersteds, the formation of high-speed electrons must take place within the surface layers, whence they can easily escape, and not within the depths. /799

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